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ABSTRACT

A planning cycle for any autonomous university entity contains five basic processes: information storage and retrieval forecasting, resource allocation, scheduling, and a term of study with a feedback loop. The resource allocation process is investigated for the development of shortrange planning models. Dynamic models wth linear and quadratic preference functions are suggested for planning teaching activities if the nee for resource planning is anticipated. On the other hand, if the ranges of resources are known, the decisionmaker can determine a plan not only for the teaching activities but also one for the allocation of resources. (Author)

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SHORT RANGE PLANNING for EDUCATIONAL MANAGEMENT

by

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INTRODUCTION

The paper considers the short range planning process which takes place at the lower level decision making nodes in the organizations of higher learning. In particular, we investigate the planning of teaching activities at the basic autonomous entities of universities. At present this basic autonomous entity appears as a department in a faculty of a university. The decision makers at this node are usually the department chairman and his assistants. These decision makers have to plan for the operational realization and implementation of teaching, research, and other support activities. With respect to teaching activities, they need to determine, the mixture of course offerings in a curriculum. This is sometimes called the curriculum design activity. Though this planning activity takes place a term in advance and usually it is prepared only for the next term of study, the departmental decision makers generally need to anticipate the teaching activities beyond the next term of study.

There are a variety of limitation on the resources. The student enrollment behavior tend to vary from one term to the next. Therefore, all the courses in the catalog of a faculty cannot be offered each term by the department. For these reasons, and others that we need not enumerate here, the teaching activities of a department ought to be planned with reliable student enrollment forecasts within fixed and anticipated amount of resources. Here, we propose teaching activity

models within a structure of mathematical programming methodologies for academic planning activities in the autonomous entities of the institutions of higher learning. These models are for short range planning. They treat the resource allocation within a frame of planning cycle.

On the basis of the Weathersby-Weinstein [5] investigation, this sort of analysis is rarely found in the literature of university planning models. The models presented here are not suggested for the problems encountered in aggregate university planning or national educational planning. We recognize the need for aggregate planning models. It is known that there are a number of interesting studies currently undertaken at that level. A number of references are available for the pursuers of the aggregate planning models.[2,5]

The planning activities of an autonomous entity in a university organization appears to have a well defined cycle. We conceptualize this cycle with five basic parts as: (1) Information Storage and Retrieval, (2) Forecasting, (3) Resource Allocation, (4) Scheduling, (5) Term of Study with a feedback loop.

We assume that: (1) There is an implemented information storage and retrieval system such that, at least, the forecasts for the teaching resources and the student enrollments can be made with reliability.[3,4] (2) Time tables can be prepared although most of the scheduling algorithms still leave much to be desired.[1] It is natural to realize that resource allocation and scheduling programs could not satisfy the demands completely and there would be spill over effects. Obviously, the results of a term of study will change the status of the students. We assume that the spill over effects and the changes in status are captured by the information system and the data base is updated appropriately in preparation for a next cycle of planning.

DYNAMICAL MODELS

We consider a particular general dynamical model for the planning of the teaching activities at the autonomous nodes of decision making. The models presented here do not exactly fit to the structure of dynamic programming. There is a master program and a subprogram for each period of planning.

Let us state the master program as:

Optimize

$$G\,[g_n(x(n)),\ b(n)]\ , \tag{1}$$

Subject to

$$b(n) = T\,[b(n-1)]\ , \tag{2}$$

for $n = 1, 2, \ldots, N$, where N represents the number of planning periods, $G[\cdot]$ is the total return function, and the vector of resources and demands, $b(n)$, $(m+k)x1$, is transferred from one period of planning to the next under the transfer rule, $T[\cdot]$.

Let the subprograms be stated in general as:

Optimize

$$g_n(x(n)) \tag{3}$$

Subject to

$$A(n)\ x(n) \leq b(n) \tag{4}$$

$$x(n) \geq 0 \text{ and integer} \tag{5}$$

The vector of unknowns, x(n), kx1, correspond to the decisions to be made for the teaching activities. There is a decision vector for each period of planning, for n = 1, 2, ..., N. The element $x_j(n)$ takes on an integer value and indicates the planned number of units for the j-th teaching activity, for j = 1, 2, ..., k.

The educational technology matrix, A(n), (m + k) xk, could vary from one period of planning to the next. However, a constant A matrix is sufficient if no changes are expected within next N periods. We assume that the model contains two basic sets of constraints. In one of these sets, the resource constraints are stated. In the other, the demand constraints are stated. The matrix A(n) is partitioned with respect to these two sets of constraints as:

$$A(n) = \begin{pmatrix} R(n) \\ \\ D(n) \end{pmatrix} \qquad (6)$$

where R(n) is mxk and D(n) is kxk.

Similarly, the right hand side vector, b(n), (m+k)x1, is partitioned with respect to these sets as:

$$b(n) = \begin{pmatrix} h(n) \\ \\ s(n) \end{pmatrix} \qquad (7)$$

where h(n) is mx1 and s(n) is kx1. The upper bounds on resource availabilities are stated in the vectors h(n). The bound on demands are stated in the vectors s(n). Here the demands are the forecasted student enrollments for each of the teaching activities. The resources are the forecasted teaching staff, space, and other support availabilities. In the context of this paper, we consider only the expected values of the demands and resources. We do not treat the stochastic aspects of these parameters.

An element $r_{ij}(n)$ specifies the required amount of the i-th resource for the j-th teaching activity in the n-th period of planning. An element $h_i(n)$ indicates the expected amount of the i-th resource in the n-th planning period. On the other hand, an element $d_{ii}(n)$ specifies the recommended class size for the i-th teaching activity. Whereas the element $s_i(n)$ indicates the expected student enrollment in the i-th teaching activity. It is clear that $D(n) = \text{diag}\,(d_{ii}(n))$, $d_{ij}(n) = d_{ji}(n) = 0$, for $i \neq j$, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, k$. We identify the collection of lectures, laboratories, seminars, etc., as the teaching activity set.

We suggest that the elements $r_{ij}(n)$ be expressed in units of hours per unit of teaching activity. A unit of a teaching activity is generally a course section, a class, a laboratory, or some other suitable measure. It is conceivable that the resource requirements can be expressed in terms of other measures. However, it is known that, the catalogs of universities indicate a time measure for the lectures and

the laboratories. These can be used readily for some of the constraints. Similar, suitable measures can be found for other resource constraints. Naturally, the availability of the particular resource is expressed in terms of the same units on the right hand side of the constraint.

The elements $d_{ii}(n)$ are expressed as the number of students per unit of activity and $s_i(n)$ as the number of students.

The resource constraints can be grouped into three sets: manpower, space, and other. Manpower resource constraints include professorial, graduate student assistant, and other instructional teaching efforts. As a minimum, there may be three constraints one for each manpower constraint. However, the professorial teaching efforts can be further separated if the decision maker can identify mutually exclusive sets of teaching activities which can be aggregated. It should be noted that, whatever the level of aggregation, the decision maker is implicitly assuming interchangability among the resource entities that are summed up in a constraint equation.

Similarly there may be at least two constraints for the space resources: one for the class rooms and one for the laboratories. However, the decision maker could identify the groups of class rooms and laboratories by size and capacity, e.g., number of seats or work benches, etc. Each such group would be determined as a particular constraint.

The third set of resource constraints are related to all other support efforts such as secreterial staff, computer time, etc.

The total return function $G[\cdot]$ is the rule for the composition of

the returns from each period. This may be a very complex rule for universities. However, we assume that G [·] is separable and additive in order to provide the most simple vehicle for the model. A return from a planning period, $g_n(\cdot)$, represents a preference function. This preference function could be different for each planning period but the decision maker could assume it to be the same for each period for the sake of simplicity. This preference function, $g_n(\cdot)$, can be expressed in many forms. Here we consider only the linear and quadratic forms.

The lineary assumption specifies the form of $g_n(x(n))$ as

$$g_n(x(n)) = \dot{c}(n)\, x(n) \tag{8}$$

where $\dot{c}(n)$ is kx1 vector and the necessary transposition is understood. The elements of $\dot{c}(n)$ are the preference values associated with the teaching activities, x(n), in the n-th period of planning. Some studies have directed effort to define c(n) carefully.[2,5]

The specific form of the dynamical model can now be stated as follows:

Optimize

$$G_n [g_n(x(n)), b(n)] = \left\{ g_n(x(n)) + G_{n-1} \right\}, \tag{9}$$

Subject to

$$b(n) = \hat{b}(n) + \left\{ b(n-1) - A(n-1)\, x(n-1) \right\}, \tag{10}$$

under additional constraints of the subprograms which are stated as:

Optimize

$$g_n(x(n)) \tag{11}$$

Subject to

$$A(n)\, x(n) \leq b(n) \tag{12}$$

$$x(n) \geq 0 \text{ and integer} \tag{13}$$

for $n = 1, 2, \ldots, N$, with the possible initial conditions $b(0) = 0$, $x(0) = 0$, and $G_0 = 0$.

Our concern here is essentially with the transfer function which is stated in equation (10). We recognize here that $\hat{b}(n)$ is the initially expected, either fixed or anticipated, amount of right hand side; whereas $b(n)$ is the amount that is determined after the transfer from the previous planning period. We need to interprete the meaning of these transfer functions within the natural behavior of educational planning spirit. The right hand side vector $b(n)$ was partitioned into two sets as $h(n)$ and $s(n)$. We ask here the implications and validity of the transfer concepts for the components of resources and for the components of student enrollments. Can the decision maker transfer unused manpower time and effort into the next period of planning? Can the decision maker transfer the class room space and capacity that have not been utilized to the next period of planning? These and other questions lead one to realize that most of the educational resources are available in fixed or anticipated amounts and that they can not be transferred from one period of planning to the next.

On the other hand, we ask whether the decision maker could transfer the demands, the student enrollments, from one term to the next. Is it conceivable and appropriate that a certain subject matter would not be taught in a given term of study and that it may be offered in the next period of planning? Of course such a transfer of student enrollment demand must be justified. It may be that professorial effort to teach the course is not available either because there is no one to

teach it or because such a person has to teach another course for which the enrollment demand is greater and it has a higher preference, etc.

It is within this interpretation of the transfer functions that the dynamic programming models gain a practical sense of applicability. Otherwise, it is natural to realize that the dynamical models would entail an expected amount of difficulty because of the dimensionality. The dimensionality of the dynamical models for the teaching activities are reduced precisely because the autonomous entities of the universities have to plan within certain bounds, e.g., the academic tenure, contracted teaching hours, predetermined or expected graduate student hours, fixed or planned physical plant capacities, expected number of enrollments, etc. The decision maker is not capable of changing the amount of the teaching resources drastically from one term to the next and the demands must be satisfied as much as possible. The changes can be made only in well defined modular units of teaching resource availabilities and student enrollment.

Therefore the equation (10) must be reconsidered separately for resources and for demands. For the demands, we assume that the student enrollment can be transferred from one term to the next. Hence

$$s(n) = \hat{s}(n) + \left\{ s(n-1) - D(n-1)\, x(n-1) \right\} \quad . \tag{14}$$

On the other hand, the transfer functions for resources can take on two possible forms. If the resources are assumed to be fixed for each term and that there is no anticipated change in the future, then

$$h(n) = \hat{h}(n) \tag{15}$$

However, if the resources can be changed, we assume that there are a finite and small number of alternatives with which the changes could occur. Then the decision maker would want to know when the changes could be made, e.g., when should a new faculty member be hired, provided there is one available in the market place when should a faculty member, without tenure be asked to look for opportunities elsewhere, when should a new laboratory be opened for instructional use.

Such inquiries can be formulated as

$$h(n) = \hat{h}_\alpha(n) \tag{16}$$

where

$$\hat{h}_\alpha = \left\{\hat{h}_\alpha(1), \hat{h}_\alpha(2) \ldots, \hat{h}_\alpha(N)\right\}$$

and

$$H = \left\{\hat{h}_\alpha \,\middle|\, \alpha = 1, 2, \ldots B\right\} .$$

That is there are a set of policy alternatives, H. Each policy alternative $\hat{h}_\alpha \in H$ is associated with a series of anticipated or planned resources for the period of planning. The number of alternative policies, B, specifies the number of times the dynamic programming model must be re-iterated from the beginning to the end.

There are, then, N linear integer programming problems with straightforward student enrollment forecasts, and that the whole set must be

re-evaluated for each of the B policy alternatives.

If the best policy is identified with α^*, then the sequence of vectors $\left\{x^*(1), x^*(2), \ldots, x^*(N)\right\}$, associated with $\hat{h}_{\alpha^*}$ in the program specifies the best plan for the teaching activities through next N periods of planning.

MODELS WITH QUADRATIC PREFERENCE FUNCTION

The quadratic form was suggested in order to minimize the differences between the student enrollment demands and the teaching activities planned in a particular term of study.[2] The form of $g_n(\cdot)$ is stated as:

$$g_n(x(n)\ ) = -p(n)\ x(n) + 1/2\ x'(n)\ Q(n)\ x(n) \tag{17}$$

and where

$$p_j(n) = 2w_j\ c_j(n)\ s_j(n) \text{ and } g_{jj}(n) = 2w_j\ c_j(n)\ d_{jj}(n)$$

with w_j as the scale factor.

The specific form of the dynamical model can now be stated as follows:

Optimize

$$G_n\ [g_n(x(n)\ ),\ b(n)\ ] = \left\{ g_n(x(n)\ ) + G_{n-1} \right\} \tag{18}$$

Subject to

$$s(n) = \hat{s}(n) + \left\{ s(n-1) - D(n-1)\ x(n-1) \right\} \tag{19}$$

$$h(n) = \hat{h}_\alpha(n)\ ,\ \hat{h}_\alpha(n) \in H \tag{20}$$

under additional constraints of the subprograms which are stated as:

Optimize

$$g_n(x(n)\ ) \tag{21}$$

Subject to

$$R(n)\ x(n) \leq h(n) \qquad (22)$$

$$x(n) \geq 0 \quad \text{and integer} \qquad (22)\ (23$$

for n = 1, 2, ..., N, with the possible initial conditions b(0) = 0, x(0) = 0, and $G_0 = 0$.

It should be noted that in this formulation the student enrollment became incorporated into the objective function. There are B dynamical model runs and the results have to be interpreted for the selection of the best policy alternative as before.

The assumption that there is a set of policy alternatives, H, presupposes the existence of an anticipated plan. The question we ask now is whether these anticipated plans can be discovered via the dynamical models proposed earlier. The answer is affirmative in the following sense. Instead of assuming that H is given, we consider a range for each $\hat{h}_i(n)$ such that $\underline{h}_i \leq \hat{h}_i(n) \leq \bar{h}_i$ and that the range, $[\underline{h}_i, \bar{h}_i]$ has a finite and small number of definable points of interest for the decision maker. Let us assume that the number of definable points of interest is e_i for $h_i(n)$ and let

$$E = \prod_i e_i \tag{24}$$

for i = 1, 2, ..., m, where E is the total points of interest. Let us further assume that there are two academic planning periods in an academic year and that the resources do not change during the year because of the academic inertia. Then the optimization algorithm for the subprograms need to be run T times where

$$T = 2 \sum_{j=1}^{t} E^j. \tag{25}$$

with t = N/2. The decision maker can now determine not only the optimal teaching activity vector set $\{x^*(n) \mid n = 1, 2, \ldots, N\}$ but also

the optimal plan $\{h^*(n) \mid n = 1, 2, \ldots, N\}$ from the $Z = E^t$ alternatives for the allocation of the resources. In Table 1, we attempt to show the nature of the complexity involved in such planning models. In this table, the number of subprogram runs, T, and the resulting number of alternatives, Z, are shown for N = 6 periods of planning under the assumption that there are two academic planning periods in an academic year, and the resources do not change during the year. It is obvious that, the resource planning programs, can be handled with reasonable amounts of computer expenditure if the alternative resource sets are up to ten and three years of academic planning is considered.

E	T	Z
1	6	1
2	28	8
3	78	27
4	168	64
5	210	125
6	516	216
7	798	343
8	1168	512
9	1638	729
10	2220	1000

Table 1

Number of Subprogram Runs

And the Number of Alternatives

For a Planning Model

Where N = 6.

Let us suppose that there exist a set of prices or costs associated with each resource type as:

$$v(n) = \{v_i(n) \quad i = 1, 2, \ldots, m\}. \tag{26'}$$

Then the budget of teaching activities for the n-th planning period can be determined as:

$$f(n) = \sum_{i=1}^{m} v_i(n)\, h_i^*(n) \tag{27}$$

where f(n) is the program budget for the n-th period. Let F(N) be the total budget of the teaching activities for the N periods of planning, then:

$$F(N) = \sum_{h=1}^{N} f(n). \tag{28}$$

Hence, the financial plan can be automatically determined once an optimal plan of teaching activities are determined as

$$\{x^*(n) \mid n = 1, 2, \ldots, N\} \tag{29}$$

and that there are associated prices for the various sorts of resources.

CONCLUSIONS

We have concentrated on the short range planning models that deal with the resource allocation model of a planning cycle for the teaching activities of a department in a university. We have proposed dynamical models for anticipated plans and a search algorithm for the discovery of an optimal plan for ranges of resource availabilities. We have noted that this is quite different from a dynamic programming model in the sense that it avoids the dimensionality problems that are born from the number of activities represented in the decision vector x(n). Dynamical nature of the models presented here are related to the alternatives or the ranges of resource decisions. The models are evaluated in two stages in the sense that there is an optimization problem related to the master program and that there are subprograms related to the period of planning where the two are tied together with a recursive formula which is dependent on the transfer of student enrollments.

At present, we are investigating the validity of these models in the light of real life educational data. Several sets of data will be utilized from various departments in different faculties. We expect to report the results in the near future.

REFERENCES

(1) Holzman, A. G., and Turkes, W. R., "Optimal Scheduling in Educational Institutions," Cooperative Research Project 1323, Department of Health, Education, and Welfare, U.S.A., 1964.

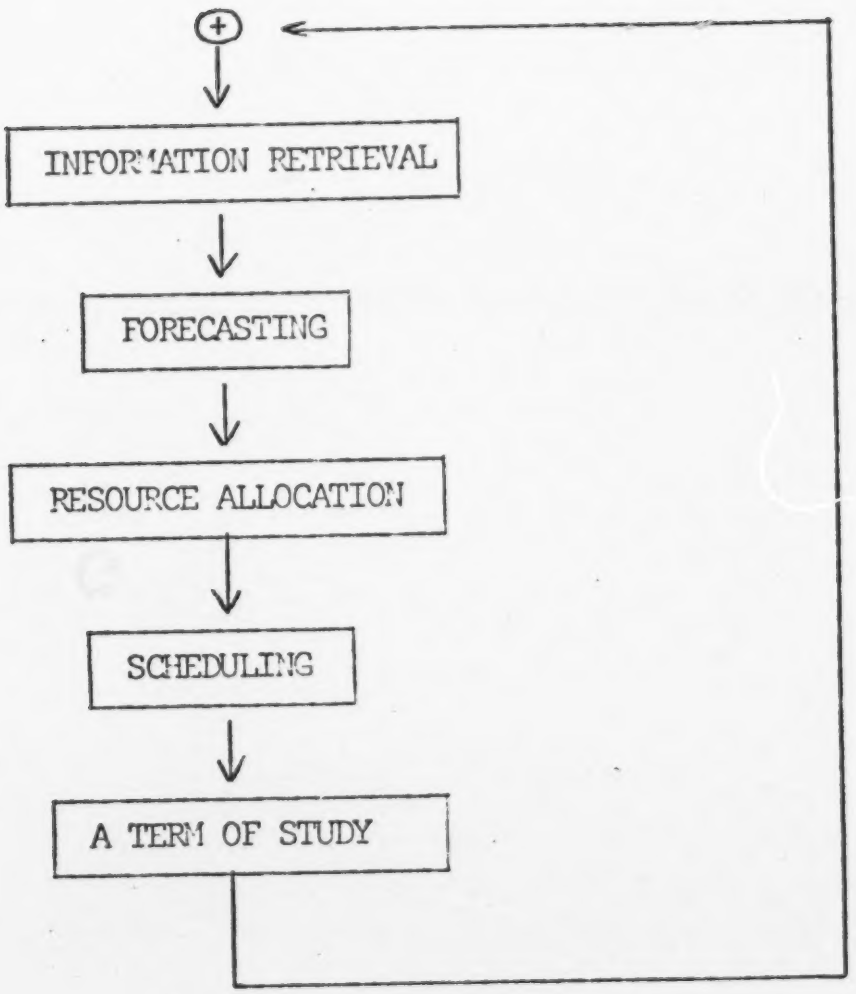
(2) Turksen, I. B. and Holzman, A. G., "Micrce Level Resource Allocation Models for Universities," 37th ORSA Meeting, April 1970.

(3) Turksen, I. B., "A Micro Level Forecasting Model of Student Enrollment," 37th ORSA Meeting, April, 1970.

(4) Turksen, I. B. and Holzman, A. G., "Basic Design Concepts and Algorithms for an Information System," CORS-CIPS Joint Conference, June, 1970.

(5) Weathersby, G. B. and Weinstein, M. C., "A Structural Comparison of Analytical Models for University Planning," TIMS National Meeting, October, 1970.

PLANNING CYCLE



DYNAMICAL MODELS

MASTER PROGRAM

OPTIMIZE

$$G\,[g_n(x(n)\,),\ b(n)]$$

SUBJECT TO

$$b(n) = T\,[b(n-1)]$$

SUBPROGRAMS

OPTIMIZE

$$g_n(x(n)\,)$$

SUBJECT TO

$$A(n)\ x(n) \leq b(n)$$

$$x(n) \geq 0 \text{ and integer}$$

FOR $n = 1, 2, \ldots, N.$

PARTITIONING

$$A(n) = \begin{pmatrix} R(n) \\ \\ D(n) \end{pmatrix}$$

$$D(n) = \text{diag}\ (d_{ii}(n))$$

$$i = 1, 2, \ldots, k$$

$$b(n) = \begin{pmatrix} h(n) \\ \\ s(n) \end{pmatrix}$$

$$R(n) = \begin{pmatrix} R_1(n) \\ R_2(n) \\ R_3(n) \end{pmatrix}$$

TRANSFER FUNCTIONS

$$b(n) = \hat{b}(n) + \left\{b(n-1) - A(n-1)\ x(n-1)\right\}$$

$$s(n) = \hat{s}(n) + \left\{s(n-1) - D(n-1)\ s(n-1)\right\}$$

ALTERNATIVE I FOR h(n)

$$h(n) = \hat{h}(n)$$

ALTERNATIVE II FOR h(n)

$$h(n) = \hat{h}_{\alpha}(n)$$

WHERE $\hat{h}_{\alpha} = \left\{\hat{h}_{\alpha}(1), \hat{h}_{\alpha}(2), \ldots, h_{\alpha}(N)\right\}$

AND $H = \left\{\hat{h}_{\alpha} \mid \alpha = 1, 2, \ldots, B\right\}$

IF α^{*} IS THE OPTIMAL POLICY

THEN $\left\{x^{*}(1), x^{*}(2), \ldots, x^{*}(n)\right\}$

IS THE OPTIMAL TEACHING ACTIVITY SET

LINEAR PREFERENCE FUNCTION

OPTIMIZE

$$G_n [g_n(x(n)), b(n)] = \left\{ g_n(x(n)) + G_{n-1} \right\}$$

SUBJECT TO

$$b(n) = \hat{b}(n) + \left\{ b(n-1) - A(n-1)\, x(n-1) \right\}$$

FURTHER SUBJECT TO

OPTIMIZE

$$g_n(x(n)) = c(n)\, x(n)$$

SUBJECT TO

$$A(n)\, x(n) \leq b(n)$$

$$x(n) \geq 0 \quad \text{and integer}$$

$$b(0) = 0,\ x(0) = 0, \text{ and } G_0 = 0,$$

FOR $n = 1, 2, \ldots, N.$

DYNAMIC PLANNING MODEL

IN STRAIGHT L. P. FORM

$$\text{MAX} \quad C = \sum_{n=1}^{N} c(n)\, x(n)$$

$$\begin{bmatrix} R(1) & & & \\ D(1) & & & \\ & R(2) & & \\ D(1) & D(2) & & \\ & & R(3) & \\ D(1) & D(2) & D(3) & \\ \vdots & \vdots & & \\ & & & R(N) \\ D(1) & D(2) & \cdots & D(N) \end{bmatrix} \begin{bmatrix} x(1) \\ \\ x(2) \\ \\ x(3) \\ \vdots \\ x(N) \end{bmatrix} \leq \begin{bmatrix} h(1) \\ s'(1) \\ h(2) \\ s'(2) \\ h(3) \\ s'(3) \\ \vdots \\ h(N) \\ s'(N) \end{bmatrix}$$

$$s'(n) = \sum_{i=1}^{n} \hat{s}(i)$$

$$D(n)\, x(n) \leq s(n)$$

$$s(n) = \hat{s}(n) + \left\{ s(n-1) - D(n-1)\, x(n-1) \right\}$$

$$n = 1, 2, \ldots, N$$

QUADRATIC PREFERENCE FUNCTION

OPTIMIZE

$$G_n\,[g_n(x(n)\,),\, b(n)] \;=\; \left\{ g_n(x(n)\,) + G_{n-1} \right\}$$

SUBJECT TO

$$s(n) = \hat{s}(n) + \left\{ s(n-1) - D(n-1)\; x(n-1) \right\}$$

$$h(n) = \hat{h}_\alpha\,(n) \quad , \quad \hat{h}_\alpha\,(n) \in H$$

FURTHER SUBJECT TO

OPTIMIZE

$$g_n(x(n)\,) = -p(n)\; x(n) + 1/2\; x'(n)\; Q(n)\; x(n)$$

WHERE $p_j(n) = 2w_j c_j(n)\; s_j(n)$

$$g_{jj}(n) = 2w_j e_j(n)\; d_{jj}(n)$$

$$g_{ij}(n) \equiv 0 \quad i \neq j$$

$$i = j = 1, 2, \ldots, k$$

$$R(n)\; x(n) \leq h(n)$$

$$x(n) \geq 0 \text{ and integer}$$

FOR $n = 1, 2, \ldots, N$, WITH $b(0) = 0$, $x(0) = 0$, AND $G_0 = 0$

PLANNING MODELS

$$t = N/2$$

$$\underline{h}_i \leq \hat{h}_i(n) \leq \bar{h}_i$$

e_i POINTS IN $[\underline{h}_i, \bar{h}_i]$

$$E = \prod_i e_i$$

$i = 1, 2, \ldots, m$ RESOURCE CONSTPAINTS

$$T = \#\text{ of RUNS} = 2 \cdot \sum_{j=1}^{t} E^j$$

$$Z = \#\text{ of PROGRAM ALTERNATIVES} = E^t$$

ASSUMPTIONS: (1) TWO PERIODS IN AN ACADEMIC YEAR

(2) NO RESOURCE CHANGES DURING THE YEAR

E	T	Z
1	2	1
2	4	2
3	6	3
4	8	4
5	10	5
6	12	6
7	14	7
8	16	8
9	18	9
10	20	10

Table 2

Number of Subprogram Runs

And the Number of Alternatives

For a Planning Model

Where N = 2.

E	T	Z
1	4	1
2	12	4
3	24	9
4	40	16
5	60	25
6	84	36
7	112	49
8	144	64
9	180	81
10	220	100

Table 3

Number of Subprogram Runs

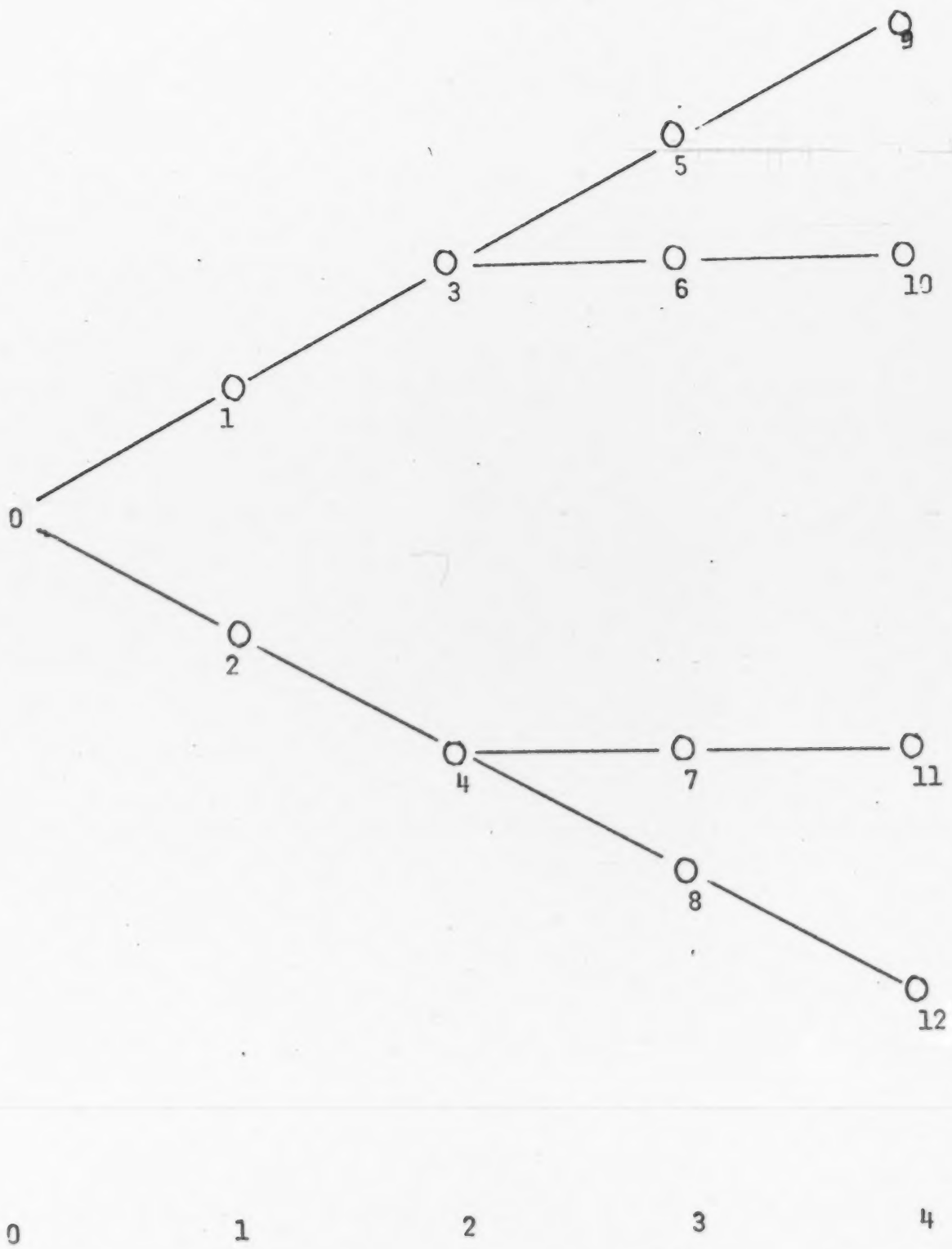
And the Number of Alternatives

For a Planning Model

Where N = 4.

FOUR STAGES (PERIODS OF PLANNING)

WITH TWO ALTERNATIVES ON RESOURCES



DYNAMIC PROGRAMMING

FORMULATION

$$G_n(s) = \text{OPTIMUM}[\, g_{sj} + G_{n-1}(j)\,]$$

s, j in

GRAPH

FOR $n = 1, 2, \ldots, N$

CONSIDER THE GRAPH

$n = 1, 2, 3, 4$ stages

$s = 1, 2, \ldots, 12$ states

$n = 1$

$g_{s\,j} + G_0(j)$

s \ j	0	j_1	$G_1(s)$
1	$g_{10} + 0$	0	g_{10}
2	$g_{20} + 0$	0	g_{20}

$n = 2$

$g_{sj} + G_1(j)$

s \ j	1	2	$J_2(s)$	$G_2(s)$
3	$g_{31}+G_1(1)$	--	1	$g_{31}+g_{10}$
4	--	$g_{42}+G_1(2)$	2	$g_{42}+g_{20}$

n = 3

$g_{sj} + G_2(j)$

s \ j	DECISION 3	DECISION 4	$j_3(s)$	$G_3(s)$
5	$g_{53} + G_2(3)$	--	3	$g_{53}+g_{31}+g_{10}$
6	$g_{63} + G_2(3)$	--	3	$g_{63}+g_{31}+g_{10}$
7	--	$g_{74} + G_2(4)$	4	$g_{74}+g_{42}+g_{20}$
8	--	$g_{84} + G_2(4)$	4	$g_{84}+g_{42}+g_{20}$

n(4)

$g_{sj} + G_3(j)$

	DECISIONS 5	6	7	8	$J_4(s)$	$G_4(s)$
9	$g_{95}+G_3(5)$	--	--	--	5	$g_{95}+g_{53}+g_{31}+g_{10}$
10	--	$g_{10,6}+G_3(6)$	--	--	6	$g_{10,6}+g_{63}+g_{31}+g_{10}$
11	--	--	$g_{11,7}+G_3(7)$	--	7	$g_{11,7}+g_{74}+g_{42}+g_{20}$
12	--	--	--	$g_{12,8}+G_3(8)$	8	$g_{12,8}+g_{84}+g_{42}+g_{20}$

$$\text{MAX} \quad 4x_1 + 3x_2 + 5x_3$$

$$3x_1 + 2x_2 + 4x_3 \leq h_1^{\alpha}(1)$$

$$3x_1 + 2x_2 + 4x_3 \leq h_2^{\alpha}(1)$$

$$6x_1 \quad + 4x_3 \leq h_3^{\alpha}(1)$$

$$20x_1 \leq s_1(1)$$

$$25x_2 \leq s_2(1)$$

$$30x_3 \leq s_3(1)$$

$$\alpha = \{ \text{I}, \text{II}, \text{III} \}$$

RESOURCES : $h(n) = \hat{h}(n)$

$$\hat{h}'(n) = \begin{pmatrix} 12 \\ 40 \\ 50 \end{pmatrix} \qquad \hat{h}''(n) = \begin{pmatrix} 24 \\ 40 \\ 50 \end{pmatrix} \qquad h'''(n) = \begin{pmatrix} 36 \\ 40 \\ 50 \end{pmatrix}$$

DEMANDS : $s(n) = \hat{s}(n) - \{ s(n-1) - D(n-1)\, x(n-1) \}$

$$\hat{s}(1) = \begin{pmatrix} 40 \\ 50 \\ 60 \end{pmatrix} \qquad \hat{s}(3) = \begin{pmatrix} 70 \\ 80 \\ 40 \end{pmatrix}$$

$$\hat{s}(2) = \begin{pmatrix} 10 \\ 30 \\ 50 \end{pmatrix} \qquad \hat{s}(4) = \begin{pmatrix} 20 \\ 40 \\ 30 \end{pmatrix}$$

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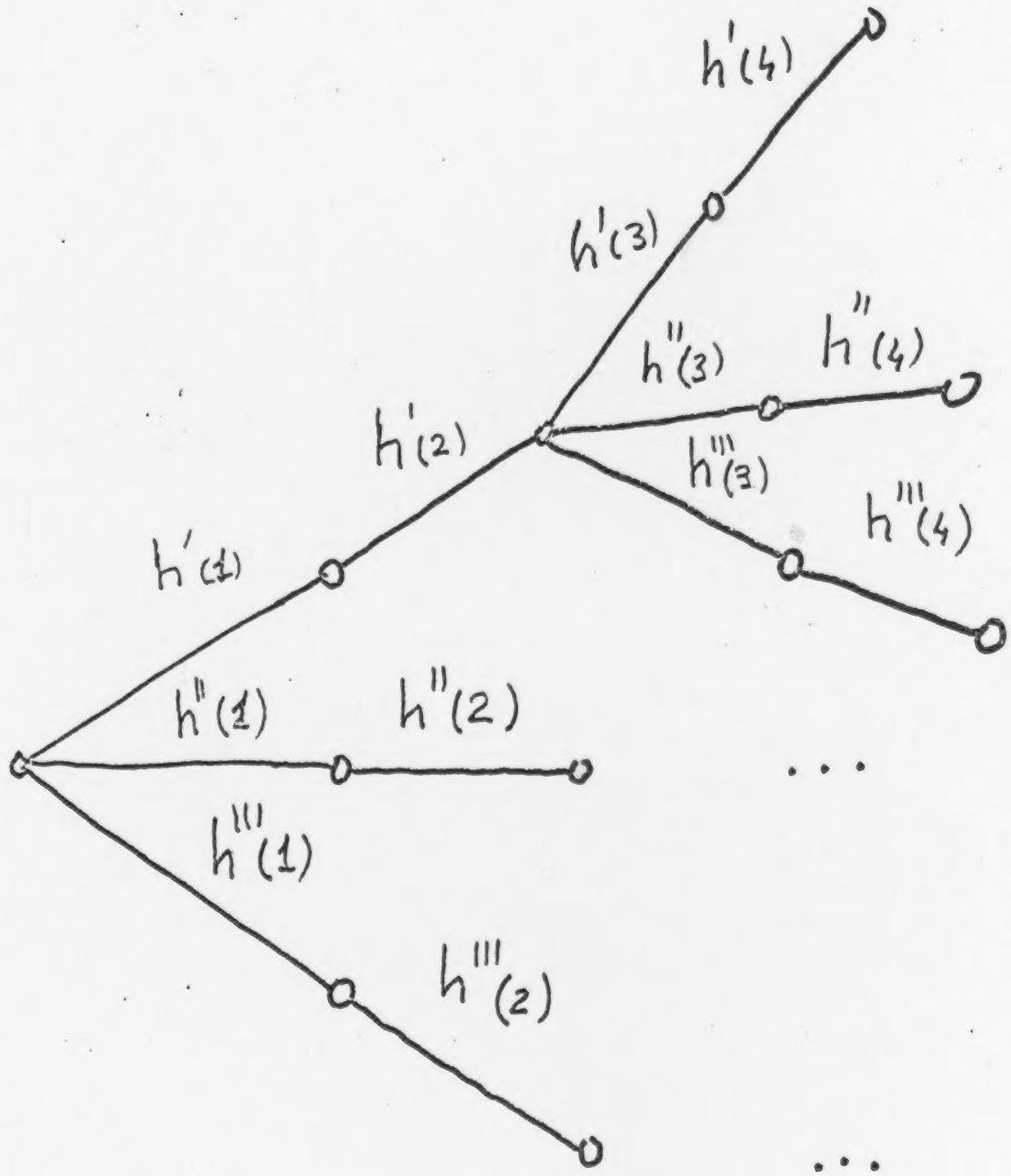

$P(z)$: ALTERNATIVE PLANS

$R(z)$: RETURNS

$C(z)$: COSTS

$$P(z) = \{ h(n,z) \} \rightarrow \{ x(n,z),\ R(z),\ C(z) \}$$

$$n = 1, 2, \cdots, N \ ; \ z = 1, 2, \cdots, Z$$

$$\underset{z \in Z_1}{\text{MAX}} \{ R(z) \} \rightarrow \underset{z' \in Z'}{P(z')} \ ; \ Z' \subset Z$$

$$\underset{z' \in Z_1}{\text{MIN}} \{ C(z') \} \rightarrow P^{*}(z)$$

EXAM

$Z_1 = \{1, 2, \cdots, 9\}$; $Z' = \{2, 3, 5, 6, 7, 8\}$

$$P^{*}(z) = P(2)$$

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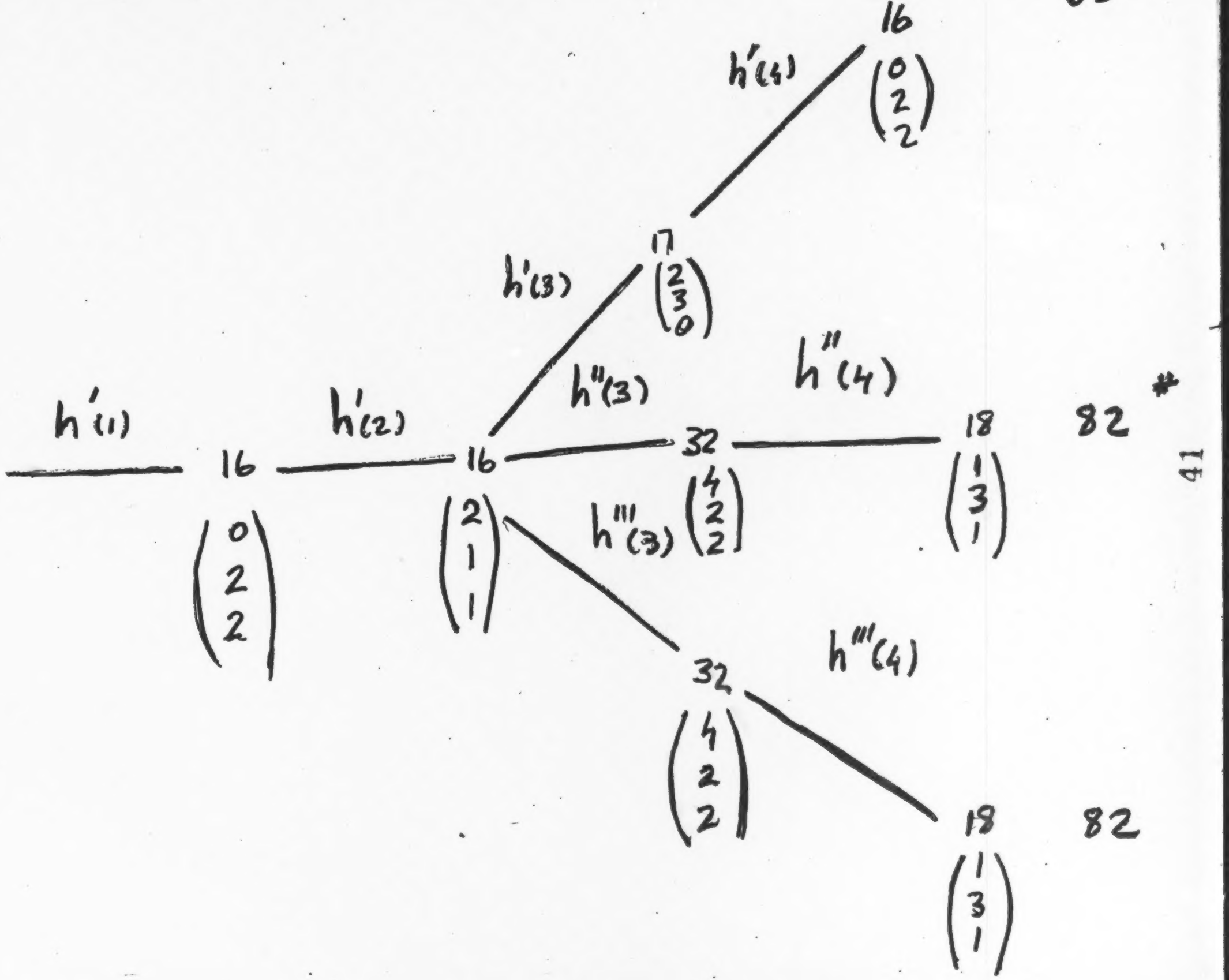
16
$\begin{pmatrix}0\\2\\2\end{pmatrix}$
h'(4)
17
$\begin{pmatrix}2\\3\\0\end{pmatrix}$
h'(3)
h''(3)
h''(4)
h'(1)
16
$\begin{pmatrix}0\\2\\2\end{pmatrix}$
h'(2)
16
$\begin{pmatrix}2\\1\\1\end{pmatrix}$
32
$\begin{pmatrix}4\\2\\2\end{pmatrix}$
h'''(3)
18
$\begin{pmatrix}1\\3\\1\end{pmatrix}$
h'''(4)
32
$\begin{pmatrix}4\\2\\2\end{pmatrix}$
18
$\begin{pmatrix}1\\3\\1\end{pmatrix}$

82 #

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